

Money and Prices: An I(2) Analysis for the Euro Area

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Abstract

The concept of integrated stochastic processes is widely used in empirical macroeconomics; and cointegration analysis is an important framework to analyze economic time series both in single equation and in system approaches. This framework is not only suited to study the relationships between variables that are integrated of order one, denoted by $I(1)$, but also to analyze variables that are integrated of higher order. However, in the literature the analysis of $I(1)$ models is much more popular than the analysis of $I(2)$ models although there is some evidence that relevant economic time series like nominal money and the price level in the euro area are integrated of order two. This is confirmed by applying tests on double unit roots.

The purpose of this paper is to illustrate the analysis of $I(2)$ variables and to show how this technique can be applied to explore the relationship between money and prices. The leading indicator property of money for prices, money demand analysis and the role of money in the transmission mechanism are addressed. It turns out that the $I(2)$ analysis provides a considerable empirical method for extracting information from monetary aggregates for monetary policy purposes.

Keywords: Double unit roots, $I(2)$ model, cointegration, euro area money demand.

JEL classification: C22, C32, E41.

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Since the concept of cointegration was introduced into the economic literature by Engle and Granger (1987), there has been a fast growing number of publications concerning the analysis of integrated and cointegrated variables. A variable is called to be integrated of order d , denoted by $I(d)$, if it has to be differenced d times to become stationary. Two or more $I(d)$ variables are cointegrated if a linear combination of these variables exists that is integrated of a lower order.¹ Most of the studies in this context have covered the analysis of $I(1)$ variables given the finding that relevant economic variables are $I(1)$.² However, the concept of integration and cointegration is more general and also well suited for the analysis of $I(2)$ variables. Moreover, some statistical evidence indicates that nominal variables like nominal money and the price level in the euro area may be integrated of order two. This paper aims at contributing to the statistical analysis and economic interpretation of models with $I(2)$ variables. Its purpose is to provide some evidence for the $I(2)$ ness of relevant macroeconomic time series and to illustrate the application of $I(2)$ models in monetary economics. The applications shown here should be interpreted as illustrations of the kind of problems and questions that can be addressed in an $I(2)$ framework. All applications discussed here are related to the role of money in the monetary policy strategy of the European Central Bank (ECB).

The statistical analysis of $I(2)$ models has been developed to a large extent by Johansen (1995b) who provides the estimation technique and asymptotic distributions of estimators and test statistics, Paruolo (1996) and Rahbek, Kongsted and Jørgensen (1999) discussing the determination of the cointegration indices and by Paruolo and Rahbek (1999) defining weak exogeneity in the $I(2)$ model. Haldrup (1998) gives an overview of the econometric analysis of $I(2)$ variables. Examples for recent applications are an $I(2)$ cointegration analysis of the purchasing power parity between Australia and the United States by Johansen (1992), the development of a structured VAR for Denmark under changing monetary regimes by Juselius (1998), a study about convergence of price indices by Juselius (1999) and an $I(2)$ cointegration analysis of small-country import price determination by Kongsted (1998).

The structure of this paper is as follows. In section 2, the formal analysis of $I(2)$ variables is presented. It is shown how the order of integration can be determined and empirical evidence for the $I(2)$ ness of money and prices in the euro area is presented. Afterwards, cointegration and error correction in models with $I(2)$ variables are explained, and estimation and testing in this framework are summarized. It is demonstrated how the cointegration indices that correspond to the cointegration rank in the $I(1)$ framework can be determined, and causality and weak exogeneity are discussed shortly. In section 3, the $I(2)$ framework is applied to analyze the relationship between money and prices in the euro area. Finally, section 4 concludes.

¹ A more formal discussion of integration and cointegration follows in section 2.

² An early and influential contribution to the analysis of non-stationary macroeconomic time series is Nelson and Plosser (1982). A standard reference for the statistical analysis of (co)integrated vector autoregressive models is Johansen (1995a). The number of applications of the $I(1)$ model is huge. An overview cannot be given here.

2.1 The Determination of the Order of Integration

2.1.1 Definition of Integrated Processes

A stochastic process $\{y_t\}_{t=1}^T$ is called integrated of order d if the differences of order d of the process, $\Delta^d y_t = (1 - L)^d y_t$, where $Ly_t = y_{t-1}$, are stationary while the process itself and differences of lower order are not stationary.³ A well known process that is integrated of order *one* is the random walk:

$$y_{1t} = \sum_{j=1}^t \varepsilon_j \quad \text{for } t = 1, \dots, T \quad \text{or} \quad y_{1t} = y_{1,t-1} + \varepsilon_t \quad \text{for } t = 2, \dots, T, \quad (2.1)$$

where $\{\varepsilon_t\}_{t=1}^T$ is a stationary, independent and identically distributed error process. The first differences of the random walk $\Delta y_{1t} = y_{1t} - y_{1,t-1} = \varepsilon_t$ are stationary. A process that is integrated of order *two* is a double sum of errors:

$$y_{2t} = \sum_{k=1}^t \sum_{j=1}^k \varepsilon_j \quad \text{for } t = 1, \dots, T \quad \text{or} \quad y_{2t} = y_{2,t-1} + y_{1t} \quad \text{for } t = 2, \dots, T. \quad (2.2)$$

Because I(2) processes are double sums, they are in general more smooth and more slowly changing than I(1) processes and a time series plot may give a first indication of the order of integration. The shape of the plotted process, however, depends crucially on the chosen parameters, especially included deterministic terms and the variance of the error process. Figure 1 shows an example of an I(1) process and an I(2) process. The data generating processes are:

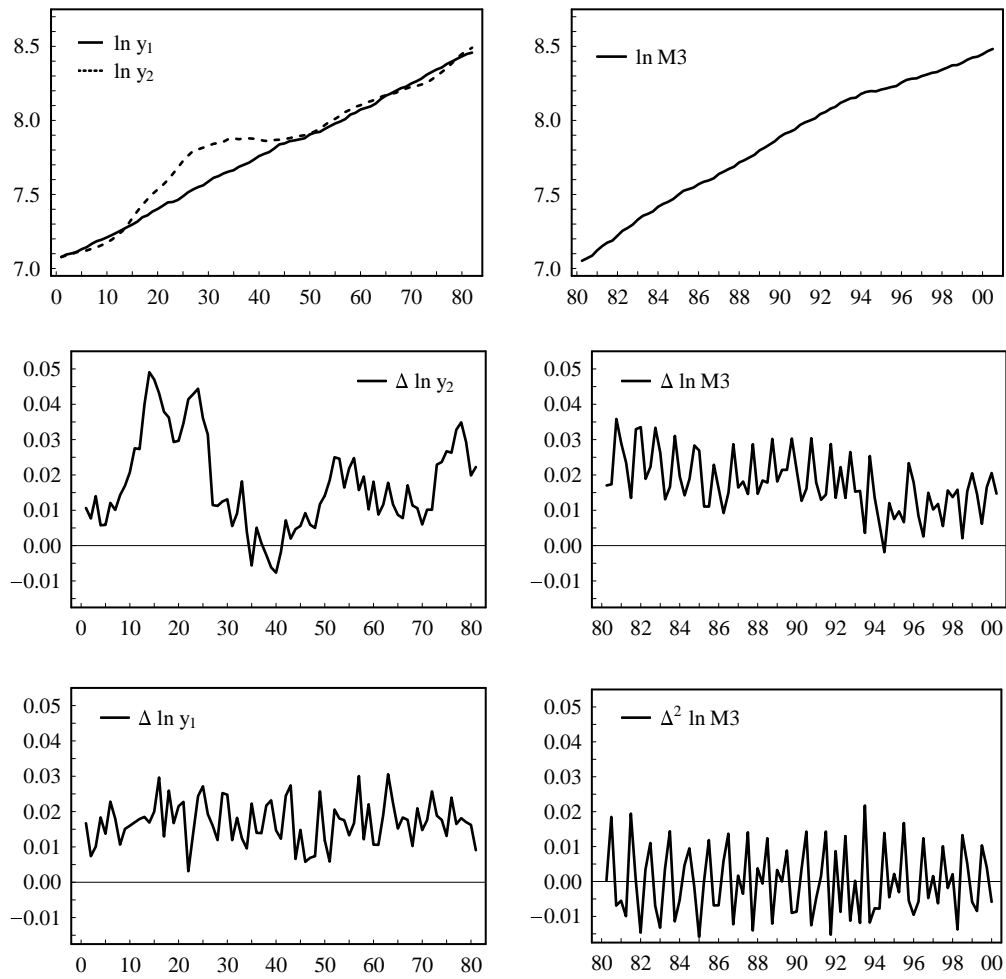
$$\begin{aligned} \ln y_{1t} &= \ln y_{1,t-1} + 0.017 + \varepsilon_{1t} \\ \ln y_{2t} &= 2 \ln y_{2,t-1} - \ln y_{2,t-2} + \varepsilon_{2t} \end{aligned}$$

with $\varepsilon_{1t}, \varepsilon_{2t} \sim N(0, 0.006^2)$, $y_{10} = y_{20} = e^{7.052}$, $y_{21} = e^{7.069}$. The variance is approximately the variance of the second differences of the logarithmic money stock M3 in the euro area from 1980 to 1999 and the starting values are the logarithmic values of M3 in the first and second quarter 1980. The processes cannot be distinguished with respect to their smoothness but the behavior of the first differences indicates the different integration orders of the processes.

The figure shows also M3 in the European Monetary Union (EMU) from 1980 to 2000, which is obviously not stationary. The money stock M3 is trending relatively smoothly, and it may be argued that it is integrated of order two for the following reasons. First, the I(2) hypothesis is supported by the inspection of the graph of the growth rates of the money stock, which are not stable and possibly not I(0) but I(1). This can also be observed for prices in the euro area. The corresponding graphs and the data description can be found in the appendix. The second differences of the logarithms of M3, that are the differences of the growth rates, are stable. Second, applying formal tests on double unit roots provides further evidence for the I(2) hypothesis.

³ A process is weakly stationary if neither its mean nor its autocovariances depend on the date t , for further information on stationarity see for example Hamilton (1994), p. 45 f.

Figure 1. Euro Area M3 and Simulations of $I(1)$ and $I(2)$ Processes y_1 and y_2



2.1.2 Tests for Double Unit Roots

Three different unit root tests are considered here:⁴ the Hasza/Fuller joint F -test (Hasza and Fuller, 1979), the Dickey/Pantula sequential t -Test (Dickey and Pantula, 1987) and the Sen/Dickey symmetric F -test (Sen and Dickey, 1987). The first difference of an $I(2)$ process is

$$\Delta y_{2t} = y_{2t} - y_{2,t-1} = \sum_{k=1}^t \sum_{j=1}^k \varepsilon_j - \sum_{k=1}^{t-1} \sum_{j=1}^k \varepsilon_j = \sum_{j=1}^t \varepsilon_j = \sum_{j=1}^{t-1} \varepsilon_j + \varepsilon_t = \Delta y_{2,t-1} + \varepsilon_t. \quad (2.3)$$

This can also be written as

$$y_{2t} = y_{2,t-1} + \Delta y_{2,t-1} + \varepsilon_t \quad (2.4)$$

and leads to the generalized Dickey-Fuller regression

$$x_t = \hat{\alpha}_1 x_{t-1} + \hat{\alpha}_2 \Delta x_{t-1} + \hat{u}_t. \quad (2.5)$$

Subtracting x_{t-1} and Δx_{t-1} from both sides of (2.5) and augmenting the regression with lagged second differences to reduce the autocorrelation of the residuals such that they become white

⁴ The following description of parametric tests for double unit roots benefits from Haldrup (1998), who also describes nonparametric tests.

$$\Delta^2 x_t = (\hat{\alpha}_1 - 1)x_{t-1} + (\hat{\alpha}_2 - 1)\Delta x_{t-1} + \sum_{j=1}^{k-2} \hat{\varphi}_j \Delta^2 x_{t-j} + \hat{\varepsilon}_t, \quad (2.6)$$

where $k \geq 2$ is the order (lag length) of the autoregressive process $x_t = \sum_{j=1}^k \alpha_j^* x_{t-j} + \varepsilon_t$ in its finite order level representation. The Hasza/Fuller joint test for double unit roots is an F -test of the hypothesis $H_0: \alpha_1 = \alpha_2 = 1$. The F -statistic⁵ is calculated from the sum of squared residuals $RSS_1 = \sum_{t=k+1}^T \hat{\varepsilon}_t^2$ from the regression (2.6) and the sum of squared residuals $RSS_0 = \sum_{t=k+1}^T \hat{\varepsilon}_t^{(0)2}$ from the restricted regression (2.7), where $\alpha_1 = \alpha_2 = 1$ is imposed,

$$\Delta^2 x_t = \sum_{j=1}^{k-2} \hat{\varphi}_j^{(0)} \Delta^2 x_{t-j} + \hat{\varepsilon}_t^{(0)}, \quad (2.7)$$

in the following way:

$$\text{HF-}F = \frac{(RSS_0 - RSS_1)/2}{RSS_1/(T - m)},$$

where $m = 2 + (k - 2) = k$ is the number of regressors in the unrestricted regression. Under the null hypothesis that x_t is integrated of order 2, the F -test statistic has a non-standard limiting distribution. The empirical critical values are tabulated in Hasza and Fuller (1979) and are reproduced in table 8, case (d), in the appendix. If the true data generating process is a stochastic process around a linear and/or quadratic time trend, then the probability of not rejecting a wrong null hypothesis tends to one, if the test statistic is constructed from the test regression (2.6). To obtain a reliable test, it is necessary to add the relevant deterministic terms to the test regressions. If the time series plot shows that the process has possibly a deterministic quadratic trend, a quadratic trend polynomial

$$\mu_t = \mu_0 + \mu_1 t + \mu_2 t^2 \quad (2.8)$$

has to be added to the test regressions (2.6) and (2.7), in this case $m = k + 3$. Equivalently, if the process exhibits possibly a linear trend like M3 in figure 1, then the linear trend polynomial

$$\mu_t = \mu_0 + \mu_1 t \quad (2.9)$$

has to be added to the test regressions (2.6) and (2.7), and now $m = k + 2$. The corresponding empirical critical values are also reproduced in table 8 in the appendix.

The Dickey/Pantula sequential test on double unit roots consists of two stages. In the first stage, the usual augmented Dickey-Fuller test (ADF) is used to test whether the first differences are $I(1)$. In this stage it is assumed that the process is at least $I(1)$, therefore it is a conditional test. The ADF test regression for the first differences is

$$\Delta^2 x_t = (\hat{\alpha}_2 - 1)\Delta x_{t-1} + \sum_{j=1}^{k-2} \hat{\varphi}_j \Delta^2 x_{t-j} + \hat{\varepsilon}_t \quad (2.10)$$

and the Dickey/Pantula statistic $DP-t$ is the usual t -statistic for the coefficient in front of Δx_{t-1} . $DP-t$ follows the Dickey-Fuller limiting distribution. The critical values are tabulated in Hamilton (1994), for example. The power of this test and the critical values depend again on the included deterministic terms. If the alternative to the unit root in the first differences is a linear time trend, then a trend has to be included in the test regression. Otherwise, the test has no

⁵ See Hamilton (1994), p. 206 f., for example.

Table 1: Unit root tests for M3, GDPP and HICP

	ln M3			ln GDPP			ln HICP		
	lags 1..4			lags 1..4			lags 1..2		
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
HF- F	4.22	4.12	6.75	4.00	4.26	9.88	3.37	6.52	13.28*
SD- F	0.45	3.04	13.52	0.34	1.28	13.17	1.39	1.88	9.89
DP- t	-2.11	-2.66	—	-1.29	-2.42	—	-2.15	-1.50	—

Notes: The test statistics reported are as described in the text, lags refers to the number of lagged differences included in the auxiliary regression. (a), (b) and (c) indicate that a constant, a constant and a linear trend or a constant, a linear trend and a quadratic trend are included. Seasonal dummies have been included in all test regressions. * refers to significance, that is rejection of the null hypothesis of a double unit root, on a 10% level. The data is described in the appendix.

asymptotic power at all. If the null hypothesis of a unit root in the first differences is rejected, then the second stage of the test, the usual ADF test for the process in levels is performed to test whether the process itself is $I(1)$.

The Sen/Dickey symmetric joint test for double unit roots is based on the estimation of the following two-equations system:

$$\begin{aligned}\Delta^2 x_t &= (\hat{\alpha}_1 - 1)x_{t-1} + (\hat{\alpha}_2 - 1)\Delta x_{t-1} + \sum_{j=1}^{k-2} \hat{\varphi}_j \Delta^2 x_{t-j} + \hat{\varepsilon}_{1t} \\ \Delta^2 x_t &= (\hat{\alpha}_1 - 1)x_{t-1} - (\hat{\alpha}_2 - 1)\Delta x_t + \sum_{j=1}^{k-2} \hat{\varphi}_j \Delta^2 x_{t+j} + \hat{\varepsilon}_{2t}\end{aligned}\quad (2.11)$$

Because of the cross-equation restrictions in (2.11), the dependent variables and the independent variables of the two equations are stacked together such that an extended Hasza/Fuller regression has to be estimated. The F -test statistic for $H_0: \alpha_1 = \alpha_2 = 1$ is the symmetric Sen/Dickey test statistic SD- F . Because the estimation is more efficient, the Sen/Dickey test has a higher power than the Hasza/Fuller test. Like the previous tests, the Sen/Dickey regression has to be augmented by the appropriate deterministic terms. The critical values are reproduced in table 9 in the appendix.

The previously described unit root tests are applied to the logarithms of M3, GDP deflator (GDPP) and the harmonized index of consumer prices (HICP) in the EMU; the results are reported in table 1.⁶ The table can be interpreted as follows. It cannot be rejected that M3, the GDP deflator and the HICP in the euro area are integrated of order two. The non-rejection is independent of the included deterministic terms. The most realistic assumption is that a linear trend is in the data implying that column (b) is relevant for HF- F and SD- F and that column (a) is relevant for DP- t .

2.2 Representation and Interpretation of $I(2)$ Models

Because of the strong evidence for $I(2)$ ness of money and prices in the EMU, it is suggested to model the relationship between these variables in an $I(2)$ framework. As it is not necessary in

⁶ The calculations have been done with Mathematica 4.0.

an I(1) model that all variables are I(1), it is not necessary in an I(2) model that all variables are I(2). Cointegration in the I(2) framework is a restriction like cointegration in the I(1) model and this restriction can be tested. In the following, the representation and interpretation of I(2) models is discussed.

Consider the following vector autoregressive model with two lags, VAR(2):

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + u_t, \quad t = 3, \dots, T, \quad (2.12)$$

where x_t is a $p \times 1$ vector of endogenous variables, A_i are $p \times p$ coefficient matrices and $u_t \sim N_p(0, \Sigma_u)$ is a $p \times 1$ vector of multivariate normally distributed errors. Subtracting x_{t-1} gives the corresponding vector error correction model (VECM):

$$\begin{aligned} \Delta x_t &= (A_1 - I_p)x_{t-1} + A_2 x_{t-2} - A_2 x_{t-1} + A_2 x_{t-1} + u_t \\ &= (A_1 + A_2 - I_p)x_{t-1} - A_2(x_{t-1} - x_{t-2}) + u_t \\ &= \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + u_t, \end{aligned} \quad (2.13)$$

with I_p as p -dimensional identity matrix. Further subtracting Δx_{t-1} gives the following representation:

$$\begin{aligned} \Delta^2 x_t &= \Pi x_{t-1} + (\Gamma_1 - I_p) \Delta x_{t-1} + u_t \\ &= \Pi x_{t-1} + \Gamma \Delta x_{t-1} + u_t. \end{aligned} \quad (2.14)$$

The process is called integrated of order d , $x_t \sim I(d)$, if the highest order of integration occurring in the set of variables in x_t is d . The process is called cointegrated of order (d, i) , $x_t \sim CI(d, i)$, if the process itself is integrated of order $d > 0$, and there exists a linear combination $\beta' x_t$ that is integrated of a lower order: $\beta' x_t \sim I(d - i)$, $i > 0$.⁷ The number of linear independent cointegration vectors equals $r = \text{rk}(\Pi)$ and β is a $p \times r$ matrix of cointegration vector(s). Cointegration implies a set of testable restrictions on the coefficients in the VECM. The process can be described by an I(1) cointegration model, $x_t \sim CI(1, 1)$, if

$$0 < \text{rk}(\Pi) = r < p \quad \text{and} \quad \text{rk}(\alpha'_\perp \Gamma \beta_\perp) = p - r, \quad (2.15)$$

where α and β are $p \times r$ matrices such that $\Pi = \alpha \beta'$ and γ_\perp denotes an $p \times (p - r)$ orthogonal matrix to the $p \times r$ matrix γ such that $\gamma' \gamma_\perp = 0$.⁸ The system is driven by $p - r$ common stochastic I(1) trends, $\beta'_\perp x_t$.

If

$$0 < \text{rk}(\Pi) = r < p \quad \text{and} \quad \text{rk}(\alpha'_\perp \Gamma \beta_\perp) = s < p - r, \quad (2.16)$$

the process is I(2) and is driven by $p - r - s$ common stochastic I(2) trends. The matrix $(\alpha'_\perp \Gamma \beta_\perp)$ can be decomposed into the two $(p - r) \times s$ matrices ξ and η such that $(\alpha'_\perp \Gamma \beta_\perp) = \xi \eta'$. The basis of the I(0), I(1) and I(2) relations can be estimated as the matrices β , $\beta_1 = \bar{\beta}_\perp \eta$ and $\beta_2 = \beta_\perp \eta_\perp$, where the bar symbolizes the operation $\bar{\gamma} = \gamma(\gamma' \gamma)^{-1}$ implying $\gamma' \bar{\gamma} = I$. A linear combination of the common $p - r - s$ I(2) trends is $\beta'_2 x_t$.⁹ The number of the I(1) relations, $\beta'_1 x_t$, is s , and there are r I(0) relations. In general, these relations contain the levels and the first differences of x_t :

$$\beta' x_t - \delta \beta'_2 \Delta x_t \sim I(0), \quad (2.17)$$

⁷ The concept of cointegration was introduced by Engle and Granger (1987).

⁸ If $r = p$, then $\gamma_\perp = 0$ and if $r = 0$, then $\gamma_\perp = I_p$ is chosen.

⁹ In equivalence to the I(1) model, where the common I(1) trends have the basis β_\perp , the basis of the common I(2) trends in the I(2) model can also be written as $\beta_2 = (\beta, \beta_1)_\perp$. For details see Haldrup (1998).

where $\delta = \alpha_1 \beta_2$ is of dimension $r \times (p - r - s)$. $\beta_2 \Delta x_t$ are the differenced I(2) trends and therefore always I(1) and not cointegrated. If $r > (p - r - s)$, the I(0) relations can be separated into two groups. First, there are $r - (p - r - s)$ relations that do not need the first differences to become stationary:¹⁰

$$\delta'_\perp \beta' x_t \sim I(0), \quad (2.18)$$

where δ_\perp is of dimension $r \times (r - (p - r - s))$. Second, there are $p - r - s$ *polynomially cointegrating* relations containing also first differences of the I(2) variables:

$$\delta' \beta' x_t - \delta' \delta \beta'_2 \Delta x_t \sim I(0). \quad (2.19)$$

Notice that the cointegration parameters of interest β , β_1 and δ are not unique such that identifying restrictions have to be imposed.

Now, the VECM can also be written in the following way:

$$\Delta^2 x_t = \alpha(\beta' x_{t-1} - \delta \beta'_2 \Delta x_{t-1}) - (\zeta_1, \zeta_2)(\beta, \beta_1)' \Delta x_{t-1} + u_t \quad (2.20)$$

with $\zeta_1 = \Gamma \bar{\beta}$ and $\zeta_2 = \Gamma \bar{\beta}_1$, see Paruolo and Rahbek (1999).

The VAR model (2.12) can be expanded to higher lag length and deterministic terms can be included:

$$x_t = \mu_t + \sum_{i=1}^k A_i x_{t-i} + u_t \quad (2.21)$$

or written as VECM

$$\Delta^2 x_t = \mu_t + \Pi x_{t-1} + \Gamma \Delta x_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 x_{t-i} + u_t, \quad (2.22)$$

with $\Pi = -I_p + \sum_{i=1}^k A_i$, $\Gamma_i = -\sum_{j=i+1}^k A_j$ ($i = 1, 2, \dots, k-1$), $\Gamma = -I_p + \sum_{i=1}^{k-1} \Gamma_i$, and $\Psi_i = -\sum_{j=i+1}^{k-1} \Gamma_j$ ($i = 1, 2, \dots, k-2$). If the coefficients of the representation in second differences are given, the corresponding level coefficients can be calculated in the following way:

$$\begin{aligned} A_k &= \Psi_{k-2} \\ A_{k-1} &= \Psi_{k-3} - 2\Psi_{k-2} \\ A_{k-i} &= \Psi_{k-i-2} - 2\Psi_{k-i-1} + \Psi_{k-i} \\ &\dots \\ A_2 &= -I_p - \Gamma - 2\Psi_1 + \Psi_2 \\ A_1 &= 2I_p + \Gamma + \Pi + \Psi_1. \end{aligned} \quad (2.23)$$

Like in the I(1) framework, there are different possibilities to include deterministic terms in the model. The choice of the trend polynomial depends on the trending behavior of the variables. An unrestricted constant, $\mu_t = \mu_0$, for example, generates a quadratic time trend in the levels. Details about the inclusion of deterministic terms and its implications can be found in the appendix.

2.3 Estimation, Testing and the Determination of the Cointegration Indices

2.3.1 The Hierarchy of the Hypotheses about the Cointegration Indices in the I(2) Model

The hypotheses about the cointegration indices r and s in the I(2) model have a certain hierarchical ordering comparable to the ordering of hypotheses about the cointegration rank r in the

¹⁰ A special case is $\beta' x_t \sim I(0)$, implying that $\delta = 0$, $\delta_\perp = I_r$ and therefore $\delta'_\perp \beta' x_t = \beta' x_t \sim I(0)$.

$p - r$	r						
p	0	$H_{0,0} \subset H_{0,1} \subset \dots \subset H_{0,p-1} \subset H_{0,p} = H_0^0 \subset H_0$					
$p-1$	1	$H_{1,0} \subset \dots \subset H_{1,p-2} \subset H_{1,p-1} = H_1^0 \subset H_1$					\cap
							\cap
							\vdots
							\vdots
							\cap
1	$p-1$	$H_{p-1,0} \subset H_{p-1,1} = H_{p-1}^0 \subset H_p$					
$p-r-s$	p	$p-1$	\dots	1	0		

I(1) framework. As has been discussed in the previous section, the number of I(0), I(1) and I(2) relations in the system is r , s , and $p-r-s$, respectively. To start with, consider the case in which $s = p-r$ such that the number of I(2) trends is zero and the model reduces to an I(1) model. In this case, the trace test on the cointegration rank is performed in the following order: The model with a cointegration rank of at most r is denoted H_r . The model, where the cointegration rank is exactly r (and not at most r) is called H_r^0 . It follows that H_0 is a subset of H_1 which is a subset of H_2 and so on: $H_0 \subset H_1 \subset \dots \subset H_p$. Testing for the cointegration rank follows the so-called Pantula principle saying that the test is consistent when the most restrictive model is tested first and the restrictions are relaxed subsequently until the unrestricted model is considered.¹¹ According to this procedure, the hypothesis that the cointegration rank is at most zero is tested first. If this is rejected, the next step is to test the hypothesis that the cointegration rank is at most 1. If this is not rejected, the cointegration rank is not at most zero and it is at most one, therefore, as a logical consequence, it is exactly one. If H_1 is rejected, the algorithm continues in an analogous way until H_r is not rejected, which indicates that the cointegration rank is exactly r . In the I(2) framework this extends to a two-dimensional order of hypotheses, see Johansen (1995b, p. 26 f.), and the model with cointegration indices of exactly r and at most s is called $H_{r,s}$. In equivalence to the I(1) case, the model with cointegration indices of exactly r and exactly s is denoted $H_{r,s}^0$, and the ordering is $H_{r,0} \subset H_{r,1} \subset \dots \subset H_{r,p-r} = H_r^0 \subset H_r$. For an overview of the ordering of models see table 2.

2.3.2 Estimation of the Cointegration Parameters and Testing Hypotheses about the Cointegration Indices

The dominating estimation technique for I(2) models is the two-step estimator developed by Johansen (1995b). The first step is to apply the reduced rank regression technique to the I(1) model H_r^0 for each possible cointegration rank $r = 0, \dots, p-1$ and calculate the corresponding estimators of the adjustment coefficients α and the long-run relations β . The second stage consists of a second reduced rank regression conditional on the respective values for (r, α, β) for each possible value of s , that is estimation of the models $H_{r,s}^0$. Now, for all possible combinations of r and s , a test statistic can be calculated. This test statistic $S_{r,s}$ is the sum of the I(1) trace test statistic Q_r and a corresponding trace test statistic for the hypothesis that the number of I(1) relations is at most s , conditional on the cointegration rank r , $Q_{r,s}$. The order of testing follows the Pantula principle, that is the most restrictive hypothesis $r = s = 0$ is tested first. If

¹¹ This is also the underlying principle of the Dickey/Pantula sequential test on double unit roots described in section 2.1.2.

this is rejected, s is increased subsequently, according to the order of hypotheses. If all combinations of $r = 0$ and $s = 0, \dots, (p - r)$ have been rejected, the test continues with $r = 1$ and $s = 0$ and so on. The first combination of r and s that is not rejected, that is the trace test statistic $S_{r,s}$ is smaller than the corresponding critical value, is accepted as cointegration indices of the model. The asymptotic distribution of $S_{r,s}$ is nonstandard but tabulated depending on included deterministic terms.

Paruolo (1996) shows that it is also possible to test for cointegration indices and deterministic terms sequentially without size distortion. This joint test follows again the Pantula principle: the most restrictive hypotheses is tested first. The test is conducted in the following way. The ordering of hypothesis in Table 2 is augmented such that the first line comprises the hypotheses $H_{0,0} \cdots H_0^0$ for the most restrictive assumption about the deterministic terms. The next rows contain the same hypotheses but with less restrictive assumptions about the deterministic terms, respectively. Then, the table is continued with the second row of table 2, that is with the hypotheses $H_{1,0} \cdots H_1^0$. This row is again followed by rows with the same hypotheses but with less restrictive assumptions about the deterministic terms, and so on. When the table is complete, the test statistics are compared to the corresponding critical values. Let $H_{r,s}^a$ denote the hypothesis $H_{r,s}$ with deterministic specification a , and $H_{r,s}^b$ the same hypothesis with the less restrictive deterministic assumption b . First the test statistic for $H_{0,0}^a$ is compared to the corresponding critical value. If the critical value is lower, then the hypothesis is rejected and the test statistic for $H_{0,0}^b$ is considered. Then the test proceeds as before, that is the test statistic for $H_{0,1}^a$ is inspected, followed by $H_{0,1}^b$, and so on. The procedure stops if a test statistic is lower than the critical value. Notice, that this procedure is only valid for the models 2 to 3.2 in table 7 on p. 22. Model 4.1 does not fit into the ordering of deterministic models with respect to restrictiveness. Details of this procedure, a treatment of the proper inclusion of deterministic terms, and the critical values can be found in Johansen (1995b), Paruolo (1996), and Rahbek et al. (1999); the computational implementation and the critical values are summarized in the appendix.

The estimators of the adjustment coefficients and the long-run relations can be taken from the two reduced rank regressions corresponding to the chosen pair of cointegration indices. These estimators are asymptotically efficient, and, if identified, (super)consistent, see Johansen (1995b) and Paruolo (2000). Hypotheses about the long-run relations can be tested like in the I(1) framework by imposing the restrictions and computing the likelihood ratio statistic which is χ^2 -distributed. Hypothesis testing in the I(2) model is for example discussed by Haldrup (1998). Two computer programs that can be used to estimate I(2) models and to test for the cointegration indices are CATS in RATS with an extension for I(2) models by Clara Jørgensen and the RATS application Malcolm by Rocco Mosconi. The results provided in this paper have been computed with Mathematica 4.0 using a cointegration package written by the author.

2.4 Causality and Weak Exogeneity in the I(2) Framework

A possible application of vector autoregressive models is the analysis of structural relationships between the variables. The usual tools for this kind of analysis are among others Granger causality tests, impulse response functions and forecast error variance decomposition. These methods are for example discussed in Lütkepohl (1993) for the case of I(0) and I(1) variables.¹² Further, it is often interesting if some of the variables in a system are weakly exogenous implying that the model can be estimated conditional on these variables to reduce complexity without losing efficiency. In a cointegrated I(1) VAR, weakly exogenous variables do not adjust to

¹² Further references for Granger causality tests in integrated vector autoregressive models are Giannini and Mosconi (1992) and Toda and Phillips (1993).

deviations from the long-run equilibrium, this can be given an economic interpretation in applications. Weak exogeneity in the context of cointegrated I(1) VARs is for example treated in Ericsson et al. (1998). In the following, testing for Granger non-causality and weak exogeneity in the I(2) model is briefly reviewed.

The concept of Granger causality can be described as follows. A variable (or vector of variables) x_t Granger causes another variable (or vector of variables) z_t , if the inclusion of the history of x , $(x_{t-1}, x_{t-2}, \dots, x_{t-k})$, improves forecasts of z , $(\hat{z}_{t+1}, \hat{z}_{t+2}, \dots)$. In the framework of vector autoregressive models, Granger causality can be formalized as follows:¹³ In a partitioned VAR of order k

$$y_t = \begin{bmatrix} z_t \\ x_t \end{bmatrix} = \begin{bmatrix} \mu_{1t} \\ \mu_{2t} \end{bmatrix} + \begin{bmatrix} A_{11,1} & A_{12,1} \\ A_{21,1} & A_{22,1} \end{bmatrix} \begin{bmatrix} z_{t-1} \\ x_{t-1} \end{bmatrix} + \dots \\ + \begin{bmatrix} A_{11,k} & A_{12,k} \\ A_{21,k} & A_{22,k} \end{bmatrix} \begin{bmatrix} z_{t-k} \\ x_{t-k} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \quad (2.24)$$

z_t is *not* Granger-caused by x_t if and only if $A_{12,i} = 0$ for $i = 1, \dots, k$. This restriction can be tested with a usual Wald test in the case of a stationary VAR. But if some or all of the variables in the vector y_t are integrated of order $d > 0$, the Wald test statistic has a nonstandard limiting distribution. This problem is considered by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996), for example. They show that it is possible to modify the Wald test such that the test statistic has again a χ^2 -distribution. First, the lag-length k of the VAR has to be specified. When this has been done, a VAR of order $k + d$ is estimated and it is tested if $A_{k+1} = \dots = A_{k+d} = 0$. This can be tested with a Wald test because not all coefficient matrices are involved. There have to be at least d unrestricted coefficient matrices to guarantee that the Wald test statistic is χ^2 -distributed, see Toda and Yamamoto (1995), section 4. Therefore, the Wald test for $A_{k+1} = \dots = A_{k+d} = 0$ is valid if $k \geq d$. The test on Granger non-causality uses only the first k coefficient matrices because it can be supposed that the other coefficients are zero. The Wald test statistic in this framework has a χ^2 -distribution because there remains again a sufficient number of unrestricted coefficient matrices: A_{k+1}, \dots, A_{k+d} . Deterministic terms can also be added, see Dolado and Lütkepohl (1996, p. 374).

Weak exogeneity is a weaker concept than Granger non-causality in the sense that weak exogeneity is a necessary but not sufficient condition for Granger non-causality, while Granger non-causality is not necessary for weak exogeneity. Weak exogeneity of a variable x_i in an I(1) VAR represented by the VECM (2.13) is given, if all adjustment coefficients α_{ij} , $j = 1, \dots, r$, in the equation for that variable are zero. Granger non-causality requires in addition to weak exogeneity that all coefficients of the first differences of the non-causing variables, stored in a submatrix of Γ_1 , are zero. Paruolo and Rahbek (1999) have extended the concept of weak exogeneity in the I(1) model to the I(2) model in the following way. Suppose that the number of possibly weakly exogenous variables is $p - m$. A necessary condition is that the number of possibly weakly exogenous variables is lower than or equal to the number of I(2) trends: $p - m \leq p - r - s$. Suppose further that the last $p - m$ variables in the system, $(x_{m+1}, \dots, x_p)'$ are the candidates for weak exogeneity. These last $p - m$ variables can be selected with the selection matrix $b = (0, I_{p-m})'$. From theorem 3.3 in Paruolo and Rahbek (1999) follows that a necessary and sufficient condition for weak exogeneity of $b'x_t$ is

$$b'\Pi = b'\Gamma = 0. \quad (2.25)$$

This condition refers to the representation (2.14). Equivalent formulations of this condition can also be found in other representations of the I(2) model. Paruolo and Rahbek (1999) also

¹³ This follows Lütkepohl (1993), section 2.3.1.

3 I(2) Analysis of Money and Prices in the Euro Area

3.1 Leading Indicator Properties of Money

3.1.1 The Leading Indicator Property and the Monetary Policy Strategy

To achieve the overriding objective of price stability, the ECB has adopted a monetary policy strategy based on two pillars.¹⁴ These two pillars are "a prominent role for money, as signalled by the announcement of a reference value for the growth of a broad monetary aggregate; and a broadly based assessment of the outlook for future price developments and the risks to price stability in the euro area as a whole." (ECB, 1999b, p. 46) The following macroeconomic criteria for the selection of a suitable monetary aggregate in the first pillar have been applied (ECB, 1999a, p. 32 f.): (1) the empirical stability of the money demand function such that a stable relationship between money and prices exists; (2) the selected monetary aggregate should be a good leading indicator for future price developments; (3) the controllability of the monetary aggregate by the central bank has to be taken into account. In the following section, 3.1.2, the second criterion, the leading indicator property of money for prices, is analyzed.

3.1.2 Methodology and Empirical Results

In the context of time series analysis, one variable x_t is a leading indicator for another variable z_t , if forecasts of z_{t+h} , $h > 0$, can be improved by including the history of x , that is $(x_t, x_{t-1}, x_{t-2}, \dots)$ in the forecast algorithm. This is exactly the definition of Granger causality, see section 2.4. To explore the leading indicator properties of the monetary aggregates M1 and M3 for prices, the causality test described in section 2.4 is applied to bivariate and multivariate systems.¹⁵ In the multivariate framework, a short-term interest rate s_t , a long-term interest rate, ℓ_t , and real GDP, y_t , are included as exogenous variables in the VAR estimation.

The true order of integration, d , of the analyzed variables is not known. In borderline cases or if different unit root tests yield different results, the supposed maximum order of integration should be used for the described procedure. As shown in section 2.1.2, the maximum order of integration of M3 and prices in the euro area is two. Therefore, it is supposed in the following that the maximum order of integration is $d = 2$, and the causality test is performed in the framework of a VAR with $k + 2$ lags. Additionally, a constant, seasonal dummies and a time trend are included. The lag length is chosen according to the following strategy: First, the lag length k suggested by the Hannan-Quinn criterion (HQ) is taken. Then it is tested if $A_{k+1} = A_{k+2} = 0$. If this is rejected, lags are added, that is k is increased, until the test does not reject this hypothesis. Table 3 shows the Wald test statistics and the corresponding p -values. The null hypothesis of Granger-noncausality is rejected if the p -value is smaller than the desired nominal significance level. While there is only poor support of the hypothesis that M3 is Granger causing prices in bivariate systems (the null hypothesis that M3 is not Granger causing the GDP deflator

¹⁴ The monetary policy strategy of the ECB is discussed for example in Holtemöller (2002).

¹⁵ The leading indicator properties of money have also been investigated in a recent study by Trecroci and Vega (2000) using a broad range of techniques. They consider bivariate and multivariate systems, real and nominal monetary aggregates and different assumptions about the integration order of the variables. Their data is seasonally adjusted while here seasonally not adjusted data and seasonal dummies are used.

Table 3: Causality Tests for Money and Prices (1980:1–1999:4)

	(z_t, x_t)			(z_t, x_t, s_t, ℓ_t)			$(z_t, x_t, y_t, s_t, \ell_t)$		
	k	$z_t \nrightarrow x_t$	$x_t \nrightarrow z_t$	k	$z_t \nrightarrow x_t$	$x_t \nrightarrow z_t$	k	$z_t \nrightarrow x_t$	$x_t \nrightarrow z_t$
$z_t = \text{HICP}$ $x_t = \text{M3}$	4	1.99	2.31	3	3.19	4.68	3	2.68	5.06
	[0.31]	[0.11]	[0.07]	[0.07]	[0.03]	[0.01]	[0.07]	[0.06]	[0.00]
$z_t = \text{HICP}$ $x_t = \text{M1}$	4	3.28	1.93	2	0.85	2.82	2	0.68	2.58
	[0.32]	[0.02]	[0.12]	[0.17]	[0.43]	[0.07]	[0.16]	[0.51]	[0.08]
$z_t = \text{GDPP}$ $x_t = \text{M3}$	6	1.51	2.00	6	1.43	0.80	5	1.35	0.92
	[0.75]	[0.19]	[0.08]	[0.56]	[0.23]	[0.58]	[0.21]	[0.26]	[0.48]
$z_t = \text{GDPP}$ $x_t = \text{M1}$	5	2.88	1.33	6	0.40	0.49	7	0.29	0.30
	[0.71]	[0.02]	[0.26]	[0.07]	[0.87]	[0.81]	[0.21]	[0.95]	[0.95]

Notes: The reported statistics are the Wald test statistics for the null hypothesis of Granger non-causality described in the text; $z_t \nrightarrow x_t$ has to be read as the hypothesis x_t is not Granger caused by z_t . The null hypothesis is rejected if the p -value (reported in brackets) is smaller than the desired nominal significance level. The p -value below k is the p -value for the Wald test statistic for $A_{k+1} = A_{k+2} = 0$. A constant, seasonal dummies and a linear time trend are included. HICP is the harmonized index of consumer prices, GDPP is the GDP deflator, s_t is a short-term interest rate, ℓ_t is a long-term interest rate, and y_t is real GDP. All variables except for the interest rates are measured in logarithms. The data is described in the appendix.

(GDPP) / the harmonized index of consumer prices (HICP) is not rejected on the 5%-level but only on the 10%-level), there is strong evidence that the broad monetary aggregate is Granger causal for the harmonized index of consumer prices if interest rates and real GDP are included in the VAR. Given that the HICP is the target variable of ECB policy and confirming the results of Trecroci and Vega (2000), M3 can be interpreted as a leading indicator for future price developments. A further result is that prices (both GDPP and HICP) are Granger causal for the narrow monetary aggregate M1 in a bivariate system. This confirms the assumption that M1 is held mainly for transaction purposes such that it has to be adjusted if prices change.

3.2 Money Demand

The analysis of money demand and its stability plays a prominent role in empirical monetary economics.¹⁶ This section aims to illustrate the I(2) analysis of a nominal money demand system. First, in section 3.2.1 the order of integration of logarithmic real balances as a linear combination of the logarithms of money and prices is explored. In section 3.2.2, a representative example of an I(1) real money demand analysis is discussed and it is shown how the inherent assumption of long-run price homogeneity can be tested within an I(2) approach. In section 3.2.3, it is shortly explained how money can play an active or a passive role in the transmission process, while in section 3.2.4 these possible roles of money are investigated empirically.

¹⁶ See Lütkepohl and Wolters (1999) for methodological, single country, and multi-country studies of money demand, and Coenen and Vega (1999), Müller and Hahn (2000), and Brand and Cassola (2000) for money demand analysis in the euro area.

Table 4: Unit root tests for HICP and GDPP deflated M3

	lags	ln (M3/GDPP)			lags	ln (M3/HICP)		
		(a)	(b)	(c)		(a)	(b)	(c)
HF- F	1	14.80***	16.08***	16.51**	3	2.79	6.40	6.22
SD- F	1	16.26***	23.11***	23.86***	3	2.29	6.87	6.68
DP- t	1	-5.47***	-5.42***	—	3	-2.37	-2.25	—

Notes: The test statistics reported are as described in section 2.1.2, lags refers to the number of lagged differences included in the auxiliary regression. (a), (b) and (c) indicate that a constant, a constant and a linear trend or a constant, a linear trend and a quadratic trend are included. Seasonal dummies have been included in all test regressions. *, **, and *** refer to significance, that is rejection of the null hypothesis of a double unit root, on 10, 5, and 1% levels.

3.2.1 The Order of Integration of Real Balances

Most studies of money demand relations focus on the logarithm of real money using an I(1) model. This procedure implies a cointegration relationship between the logarithms of nominal money and prices, $(m_t, p_t) \sim CI(2,1)$, with cointegrating vector $\beta_1 = (1, -1)$, given that nominal money and prices are found to be I(2). This cointegration relation can be interpreted as long-run price homogeneity. The suitability of this restriction is tested in the following. Table 4 shows the results of the unit root tests described in section 2.1.2 for two different definitions of real money. First, M3 has been deflated with the GDP deflator (GDPP), and second M3 has been deflated with the harmonized index of consumer prices (HICP). While GDPP deflated money is I(1), HICP deflated money is I(2). Therefore the cointegration restriction seems reasonable if GDPP deflated money is used in money demand studies as it is almost always the case. The fact that HICP deflated money is not I(1) does not exclude a cointegration relationship between M3 and HICP, but it rejects the hypothesis that the cointegration vector is $\beta_1 = (1, -1)$.

3.2.2 Money and Prices in a Money Demand System

In an actual study, Brand and Cassola (2000) estimate an I(1) money demand system for the euro area. This study is taken as an example for the discussion. The extension from an I(1) model to an I(2) model could be demonstrated equivalently with another real money demand study, but this study is the most recent one for the euro area. The data set consists of logarithmic real balances (M3 in the euro area deflated with the GDP deflator), inflation (first differences of logarithmic GDP deflator), logarithmic real GDP, a short-term interest rate and a long-term interest rate: $x'_t = (m_t - p_t, \Delta p_t, y_t, s_t, \ell_t)$. In a vector error correction framework for I(1) variables, Brand and Cassola are able to identify three cointegration relations by imposing (over)identifying restrictions on the cointegrating vectors. These restrictions are motivated by an economic model. The main features of this model are a constant returns to scale production function in labor and capital stock, the Fisher inflation parity and a demand function for real balances. The identified cointegration vectors of the model are the Fisher inflation parity, an equation for the term structure of interest rates (expectations hypothesis) and the demand function for real balances.

As mentioned in the previous section, if real balances are modeled as endogenous I(1) variable, long-run price homogeneity is imposed. It is not in question that this is a reasonable and widely accepted assumption in economic theory, but it is an empirical question if this is a not rejected

Table 5: Trace test for the Brand/Cassola Money Demand System

r	$S_{r,s}$					Q_r
0	350.52	256.81	207.65	175.24	157.59	148.85
1		218.97	146.53	97.47	74.84	67.61
2			121.26	74.20	42.82	36.00
3				63.99	22.22	14.05
4					28.74	5.77
$p - r - s$	5	4	3	2	1	0

Notes: Test statistic $S_{r,s}$ for testing hypotheses about the cointegration indices as described in section 2.3.2. The deterministic trend polynomial follows model 3.1 in table 7. Critical values can be found in table 11 in the appendix, the lag length is two, and the sample is 1980:1-1999:4. Seasonal dummies have been added.

restriction in the empirical model. The validity of this restriction can be tested in an I(2) framework. Given that money and prices are I(2) variables and that real GDP and the interest rates are I(1) variables, the test on long-run price homogeneity can be performed in the following vector autoregressive setup: the endogenous variables are logarithmic nominal M3, the logarithmic GDP deflator, logarithmic real GDP, a short-term interest rate, and a long-term interest rate: $x'_t = (m_t, p_t, y_t, s_t, \ell_t)$. The data used here is an updated version of the data in Brand and Cassola (2000). The optimal lag length of the VAR, suggested by the Hannan-Quinn criterion, is two and the deterministic trend polynomial is chosen such that there is a non-zero mean in the I(0) components and a linear trend in the I(1) and I(2) components (model 3.1 in table 7 in the appendix). The cointegration indices are determined as $r = 3$ and $s = 1$, see table 5.

Juselius (1998) describes the following economic scenario for this combination of cointegration indices: With $r = 3$ and $s = 1$, the number of stochastic I(2) trends in the system is $p - r - s = 5 - 3 - 1 = 1$. This implies that the two I(2) variables m_t and p_t have the same I(2) trend and are cointegrated, $(m_t, p_t) \sim \text{CI}(2,1)$. Long-run price homogeneity is given if $m_t - p_t \sim \text{I}(1)$, such that the cointegrating vector is $(1, -1)$. Additionally, there are $r - (p - r - s) = 2$ relations that cointegrate directly to I(0) and $p - r - s = 1$ polynomially cointegrating relation that comprises also first differences of an I(2) variable. The directly cointegrating relations could be a money demand function or inverse velocity ($m_t - p_t - y_t$) and the term structure of interest rates ($\ell_t - s_t$). Candidates for the polynomially cointegrating relation are a short run Phillips curve ($y_t - \beta_{yt}t - \beta_{yp}\Delta p_t$, implying another trend polynomial that allows for trends in the I(0) relations) or the Fisher inflation parity ($\ell_t - 4\Delta p_t$). The last two relations are polynomially cointegrating because a differenced I(2) variable, Δp_t , is needed to get an I(0) relation.

The long-run price homogeneity can be tested by imposing the restriction $(1, -1, \cdot, \cdot, \cdot)'$ on the $r = 3$ cointegration vectors and testing this restriction within the two-step estimation framework by comparing the maximized likelihood functions of the restricted and the unrestricted model (likelihood ratio test). The restrictions have to be imposed in the first and in the second step of the estimation procedure, such that two LR test statistics have to be calculated.¹⁷ The LR test statistic in the first step is $\chi^2(3)$ -distributed and has a value of $Q_{b1} = 30.28$ with p -value 0.00. The LR test statistic in the second step has a value of $Q_{b2} = 1.26$ and is $\chi^2(1)$ -distributed (p -value: 0.26). The first step restriction is rejected at all usual significance levels. This suggests that the underlying data set or the model specification is not appropriate to estimate an

¹⁷ The details of this test are described in the appendix.

I(1) demand system for real balances in the euro area (1988–1999). It should be mentioned that the imposed restriction in the long-run price homogeneity test is different from the imposed restrictions in the real money demand system. Nominal money and the price level do not appear in each long-run relation in the money demand system. But the test of restrictions on single cointegration vectors can only be applied if all cointegration vectors are identified. The identification of all cointegration vectors in the I(2) money demand setup is not clear so far and is left for future research.

In the economic model used by Brand and Cassola, money is completely demand determined and passive. As a passively endogenous variable, the quantity of money does not play any role in the transmission mechanism of monetary policy. Especially, the money stock is not a determinant of the price level. Money is only an indicator for the stance of monetary policy conducted via the interest rate and is therefore correlated with future inflation but it does not cause future inflation and inflation is in this sense not a monetary phenomenon. According to this economic setup, there is no explicit long-run relation between money and prices in the econometric model; but besides long-run price homogeneity there is an indirect relation between money and inflation via the long-term interest rate that is included in the money demand relation and in the Fisher inflation parity relation.

A relation between money and prices that allows for a more active role of the quantity of money in the transmission process and that considers causality from money to prices in the long-run could be established in an I(2) analysis. The analysis of the error correction mechanism of the CI(2,1) relation between money and prices can be useful for investigating the leading indicator property of money for prices. Moreover, from $(m_t, p_t) \sim \text{CI}(2,1)$ follows $(\Delta m_t, \Delta p_t) \sim \text{CI}(1,1)$.¹⁸ The cointegration relation between the growth rates is called a medium-run steady-state relation by Juselius (1999).

3.2.3 The Role of Money in the Transmission Process

Following Laidler (1999), the role of endogenous but active money in the transmission process of monetary impulses could be as follows: Consider a decreasing interest rate, for example as a result of a monetary policy action. Suppose that the interest rate elasticities of money demand and loan demand are different. This is a realistic assumption because people do not borrow to hold more money but to spend it. If consumption follows the permanent income hypothesis, for example, then consumption will increase, if the interest rate decreases. Investment decisions of firms depend on the interest rate on loans. If borrowing from the bank is cheaper due to a decreasing interest rate, then investment expenditures increase. In practice, the additional liquidity achieved by borrowing from the bank appears as a liability of the bank in the deposits of the borrowing consumers and firms. Therefore, as a consequence of the balance sheet restriction, the quantity of money increases without the demand for money increasing by the same quantity. Money supply is higher than money demand and the result is a money overhang (or excess money supply). One possibility is that this money overhang is spent such that aggregate demand increases. The additional aggregate demand affects output and prices, implying that money is active, that is, helps to cause inflation. Output and prices are arguments of nominal money demand such that the money overhang vanishes or is reduced. Another possibility is that people use their excess money holdings to lower their indebtedness implying that the quantity of loans and the nominal money stock adjust until nominal money supply and desired money holdings are in equilibrium. In this case nominal money decreases and money is passive. The empirical question, whether the adjustment in the holding of real balances is done by price

¹⁸ If $\beta'_1 x_t \sim \text{I}(1)$, then $\Delta(\beta'_1 x_t) = \beta'_1 \Delta x_t \sim \text{I}(0)$.

changes (nominal money is exogenous), by nominal balances (money is passively endogenous) or by both variables (intermediate case) is now addressed in an I(2) framework.

3.2.4 Money and Prices in an I(2) Model

The best method to explore the relationship between money and prices would be a fully specified I(2) model. Unfortunately, this is much more complicated than the specification and estimation of an I(1) model. Therefore, a simpler approach is used here to demonstrate the usefulness of the I(2) framework for the discussion of the impact of money on prices.

Cointegration tests in the I(2) model can also be conducted in a single equation framework. The procedure is equivalent to the I(1) case and tests if the residuals from a regression are integrated or stationary. Cointegration is given, if the integration order of the residuals is lower than the maximum order of integration of the variables in the regression equation. If I(2) variables are involved in the regression, the residuals can be I(0), I(1) or I(2). For money and prices we can assume that they cointegrate at least to an I(1) relation, see section 3.2.1. In a regression from money on prices, the linear combination (1, -1) is not imposed and the variables possibly cointegrate to an I(0) relation. To test the hypothesis $(m_t, p_t) \sim CI(2,0)$ the following regression has been estimated:¹⁹

$$m_t = \mu_t + \beta_p p_t + u_t, \quad (3.1)$$

where μ_t includes a constant and seasonal dummies. The augmented Dickey-Fuller (ADF) t -statistic for the residuals u_t is -1.77 (with 4 lagged differences).²⁰ This value is not significant at any usual significance level. Therefore, the null hypothesis of a unit root in the residuals cannot be rejected, and money and prices do not directly cointegrate to an I(0) relation. In the same way it can be tested if money and prices cointegrate polynomially to an I(0) relation by adding Δp and/or Δm to (3.1). Without reporting the details here, no indications for polynomial cointegration of money and prices have been found.

The (m_t, p_t) -system is now extended by adding the I(1) variable logarithmic real GDP y_t . As in the I(1) model, the single equation approach is only valid, if there is only one cointegration relation. In the case analyzed here, it can be excluded that the cointegration rank r is larger than one. The number of variables is $p = 3$. It has already been confirmed that $(m_t, p_t) \sim CI(2,1)$, such that the number of I(2) trends in the system is $p - r - s = 1$, and the number of I(1) relations is at least one, $s \geq 1$. It follows that $p - 1 - s = r \leq 1$. If there is a single cointegration relation implying $r = 1$ and $s = 1$, it is a polynomially cointegration relation (the number of polynomially cointegrating relations is $p - r - s = 1$, the number of direct cointegration relations is $r - (p - r - s) = 0$). The polynomially cointegration relationship is assumed to be²¹

$$m_t = \mu_t + \beta_p p_t + \beta_y y_t + \beta_{\Delta p} \Delta p_t + u_{2t}. \quad (3.2)$$

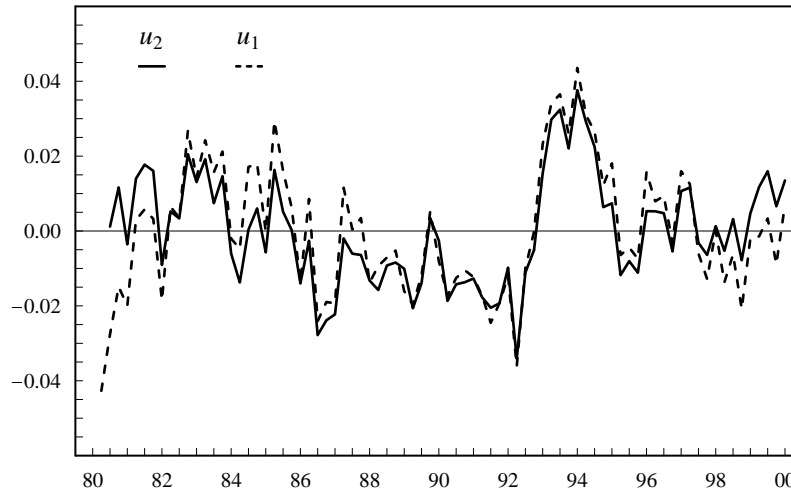
This relation can be interpreted as a nominal money demand function. Interest rates or an interest rate spread could also be added to the system, but the analysis is kept as simple as possible here. Moreover, this specification is equivalent to a specification with a long run interest rate instead of the inflation rate if the long run interest rate and the inflation rate cointegrate

¹⁹ EViews has been used to conduct the calculations in this section.

²⁰ The critical values for this cointegration test are not the usual ones, they depend on the deterministic terms, the number of I(1) regressors and the number of I(2) regressors in the cointegration regression. Single equation cointegration tests with I(2) variables are discussed in Haldrup (1994a). The critical values can also be found there.

²¹ Statistically, one could also include the first differences of the other variable that is integrated of order two, that is the money stock. But from an economic point of view, it is more meaningful to consider the inflation rate.

Figure 2: Money Overhang in I(2) Model (3.4), u_{2t} , and in I(1) Model (3.6), u_{1t}



(this is called Fisher inflation parity above) like in the Brand/Cassola model. The residuals of the money demand function are the empirical counterpart of the money overhang described in section 3.2.3:²²

$$u_{2,t-1} = \underbrace{\underbrace{m_{t-1}}_{\text{nominal money stock}} - \underbrace{(\beta_p p_{t-1} + \beta_y y_{t-1} + \beta_{\Delta p} \Delta p_{t-1})}_{\text{money demand}}}_{\text{error correction term or money overhang}} - \mu_t \quad (3.3)$$

The OLS estimation results for (3.2) are

$$m_t = \mu_t + \underbrace{1.15 p_t}_{(0.04)} + \underbrace{1.16 y_t}_{(0.06)} - \underbrace{0.24 \cdot 4 \Delta p_t}_{(0.50)} + u_{2t} \quad (3.4)$$

with OLS standard errors in parentheses, μ_t includes again a constant and seasonal dummies. The ADF t -statistic for the hypothesis that u_{2t} is integrated of order one is -4.07 (with the fourth lagged difference included), such that this hypothesis is rejected at the 10%-level (5% CV: -4.25 , 10% CV: -3.93), and it can be assumed that (3.4) is a polynomially cointegrating relation. Now, the money overhang is u_{2t} . The error correction terms of the I(2) model (3.2) and the I(1) model

$$(m - p)_t = \mu_t + \beta_y y_t + \beta_\ell \ell_t + u_{1t} \quad (3.5)$$

with OLS estimates

$$(m - p)_t = \mu_t + \underbrace{1.38 y_t}_{(0.03)} - \underbrace{0.30 \ell_t}_{(0.14)} + u_{1t} \quad (3.6)$$

are compared in figure 2. It can be seen that the money overhang is very similar in both models over long periods. At the actual end of the graph, however, it is observed that the money overhang estimated in the I(2) framework is larger than the money overhang estimated in the I(1) framework. This larger money overhang seems to be compatible with the observation that the growth rate of M3 has been clearly above its reference value in 1999.

²² In the terminology of the ECB, this is the *monetary overhang/shortfall*. The ECB uses also the nominal money gap, calculated on the basis of nominal money growth above/below the reference value, and the real money gap, that is the deviation of the observed real money stock from estimated real money demand. The ECB terminology is explained in ECB (2001).

Table 6: Summary Statistics for the Regressions (3.7) and (3.8)

Sample: 1981:2 - 1999:4 ($T=75$)					
(3.7)			(3.8)		
α_m	-0.1758	(0.0493)	α_p	0.0613	(0.0349)
\overline{R}^2	0.8343		\overline{R}^2	0.9782	
SE	0.0040		SE	0.0028	
JB	1.7372	[0.4195]	JB	3.4153	[0.1813]
LM(1)	2.2268	[0.1356]	LM(1)	3.1429	[0.0763]
LM(4)	6.8002	[0.1468]	LM(4)	8.3207	[0.0805]
ARCH(1)	0.0353	[0.8509]	ARCH(1)	1.1138	[0.2913]
ARCH(4)	1.9025	[0.7537]	ARCH(4)	16.0451	[0.0030]

Notes: A constant and seasonal dummies have been added as well as lagged differences according to equations (3.7) and (3.8), but only the estimators (standard errors in parentheses) of α_m and α_p are reported. SE denotes the standard error of the regression, JB the Jarque-Bera test for normality, LM(k) the Lagrange multiplier test for serial correlation of the residuals (k lagged residuals included) and ARCH(k) the Lagrange multiplier test for autoregressive conditional heteroskedasticity, p -values in brackets.

The error correction mechanism in the I(2) model can be captured by estimating the following two single equations:

$$\Delta^2 m_t = \mu_{mt} + \alpha_m u_{2,t-1} \quad (3.7)$$

$$+ \sum_{i=1}^3 \Phi_{11i} \Delta^2 m_{t-i} + \sum_{i=1}^3 \Phi_{12i} \Delta^2 p_{t-i} + \sum_{i=1}^4 \Gamma_{13i} \Delta y_{t-i} + u_{mt}$$

$$\Delta^2 p_t = \mu_{pt} + \alpha_p u_{2,t-1} \quad (3.8)$$

$$+ \sum_{i=1}^3 \Phi_{21i} \Delta^2 m_{t-i} + \sum_{i=1}^3 \Phi_{22i} \Delta^2 p_{t-i} + \sum_{i=1}^4 \Gamma_{23i} \Delta y_{t-i} + u_{pt}.$$

The inclusion of three lagged second differences of money and prices and four lagged first differences of real income corresponds to a lag length of five in the level specification, this lag length is suggested by the Hannan-Quinn criterion. The system (3.7/3.8) is of course not an equivalent specification to the representation (2.14) of the I(2) model. The error correction mechanism for the CI(2,1) relationship between money and prices and the third equation with the second differences of the GDP as dependent variable are omitted. But the two equations have stationary residuals and the summary statistics in table 6 do not indicate departures from the usual assumptions about the residuals (except for the ARCH(4) LM test for the price equation). The adjustment parameters $\alpha_m = -0.18$ (0.049) and $\alpha_p = 0.06$ (0.035) have correct signs and give a first impression of the adjustment process (OLS standard errors in parentheses). It can be seen that the nominal money stock is adjusting to a larger extent than the price level. This result should be interpreted carefully because the asymptotic distribution of the estimators in this kind of two-step procedure is not known. The intermediate case mentioned above with nominal money *and* prices as adjusting variables seems to be the one that is in line with the data in the euro area. The positive impact of the money overhang on future inflation is also found in Trecroci and Vega (2000) and Gerlach and Svenson (2000) using different specifications. The adjustment of nominal money can in principle be calculated in an I(1) model with real balances as endogenous variable, too. But the information is more clearly revealed in the

I(2) model without imposing *a priori* a cointegration relation between money and prices. The economic results of the I(2) analysis are compatible with the results of other money demand studies for the Euro area, for example Coenen and Vega (1999), Müller and Hahn (2000), and Brand and Cassola (2000). In all these studies, a stable money demand function is found for the Euro area. This result seems to be very robust, given that it holds for different time spans, different aggregation methods, and different statistical frameworks. Therefore, the existence of a stable money demand function, that is one of the criteria used by the ECB to chose a monetary aggregate in the first pillar (see section 3.1.1), can be confirmed for the broad monetary aggregate M3.

4 Conclusions

This paper investigates the I(2) methodology as an extension of the widely used I(1) methodology. It has been shown that there is empirical evidence that relevant macroeconomic time series like nominal money stock and price level in the euro area are integrated of order two. The concepts of causality, exogeneity, cointegration and error correction have been discussed in the I(2) framework. These concepts have been applied to test the leading indicator property of money for prices in the euro area and to illustrate the basic outline of an I(2) model of money demand.

The advantages of the I(2) approach compared to the I(1) approach in the presented applications are the following: First, relevant monetary variables in the euro area can be assumed to be integrated of order two. Therefore, an I(2) model is the appropriate statistical specification. As there is a loss of information if the first differences of variables that are integrated of order one are analyzed in an I(0) model, there is also a loss of information if differenced I(2) variables are modeled in an I(1) model, for example inflation instead of prices. Second, in the I(2) model, long-run relations between the first differences can be specified. In many applications, the logarithms of the endogenous variables are considered such that the first differences are the growth rates. There is some evidence from economic theory, that a long-run relation between the growth rate of money and the growth rate of prices (inflation) exists. This relation could be properly specified in an I(2) model. Another advantage of the I(2) model is that money and prices can be included as separate variables instead of real balances. The usual restriction of price homogeneity underlying the use of real balances can be tested in the I(2) framework. Additionally, the adjustment of nominal money to the long-run equilibrium can be seen directly from the I(2) specification without constructing it from real balances and prices.

The advantages of the I(2) model are accompanied by the following disadvantage. The instruments for the I(2) analysis are not as far developed as for the analysis of I(1) models. While the focus here has been to investigate the long-run relationships and the adjustment process, structural analysis, especially the calculation and interpretation of impulse response functions, is not discussed here. However, an example of impulse response analysis in an I(2) model can be found in Holtemöller (2002).

In summary, the I(2) model provides a considerable empirical method for extracting information from monetary aggregates for monetary policy purposes. The result of the decomposition of changes in real balances is that nominal money reacts to a larger extent than prices to deviations from the long run equilibrium of money supply and money demand. This implies that an excess money supply does not necessarily cause higher inflation in the future because this excess money supply will partly be reduced by a decreasing nominal money stock. An interesting question that has to be addressed in further research is whether it can be identified under what conditions nominal money adjusts and under what conditions prices adjust.

The Euro area data used in this paper was provided by Nuno Cassola. Further notes on the construction of the euro area aggregates (11 countries) before the introduction of the euro (back to 1980) can be found in Brand and Cassola (2000). The definition of the variables is as follows (adjusted stocks are constructed with flow statistics):

M1: Adjusted stock of the euro area monetary aggregate M1 in billions of euro. The quarterly data are averages of monthly data. The index of stocks (December 1998 = 100) is multiplied by the December 1998 stock of M1. The percentage change between any two dates (after October 1997) corresponds to the change in the aggregate excluding the effect of reclassifications etc. M1 includes currency in circulation and overnight deposits.

M3: Adjusted stock of the euro area monetary aggregate M3 in billions of euro. The quarterly data are averages of monthly data. The index of stocks (December 1998 = 100) is multiplied by the December 1998 stock of M3. The percentage change between any two dates (after October 1997) corresponds to the change in the aggregate excluding the effect of reclassifications etc. M3 comprises M1 and in addition, deposits with agreed maturity up to two years, deposits redeemable at notice up to three months (M2) and marketable instruments issued by the MFI sector (repurchase agreements, money market fund shares, money market paper and debt securities up to two years).

Real GDP (Y^r): National series on real gross domestic product at market prices are added after they have been rebased to a common base year (1995) and converted to euro via the irrevocable fixed conversion rates of 31 December 1998. Adjusted for German unification. Billions of euro.

Nominal GDP (Y^n): National series on nominal gross domestic product at market prices are added after they have been converted to euro via the irrevocable fixed conversion rates of 31 December 1998. Adjusted for German unification. Billions of euro. ESA95 data to the widest extent possible.

GDP Deflator (GDPP): Nominal GDP divided by real GDP.

HICP: Index for euro-11 area Harmonized Index of Consumer Prices. The national HICP series have been extended backwards using growth rates of CPI (national definition). Weights: consumer spending weights at purchasing power parity exchange rates in 1995. Quarterly data are averages of monthly data.

Short-term interest rate: Weighted average of euro-11 annual short-term interest rates. Weights: GDP weights at purchasing power parity exchange rates in 1995. National series are from the Bank for International Settlements, 3-month money market interest rates linked to EURIBOR from 1999 onwards. Fractions.

Long-term interest rate: Weighted average of euro-11 annual long-term interest rates. Weights: GDP weights at purchasing power parity exchange rates in 1995. National series are from the Bank for International Settlements, 10-year government bond yield or closest substitute. Fractions.

Figure 3: Euro Area Data

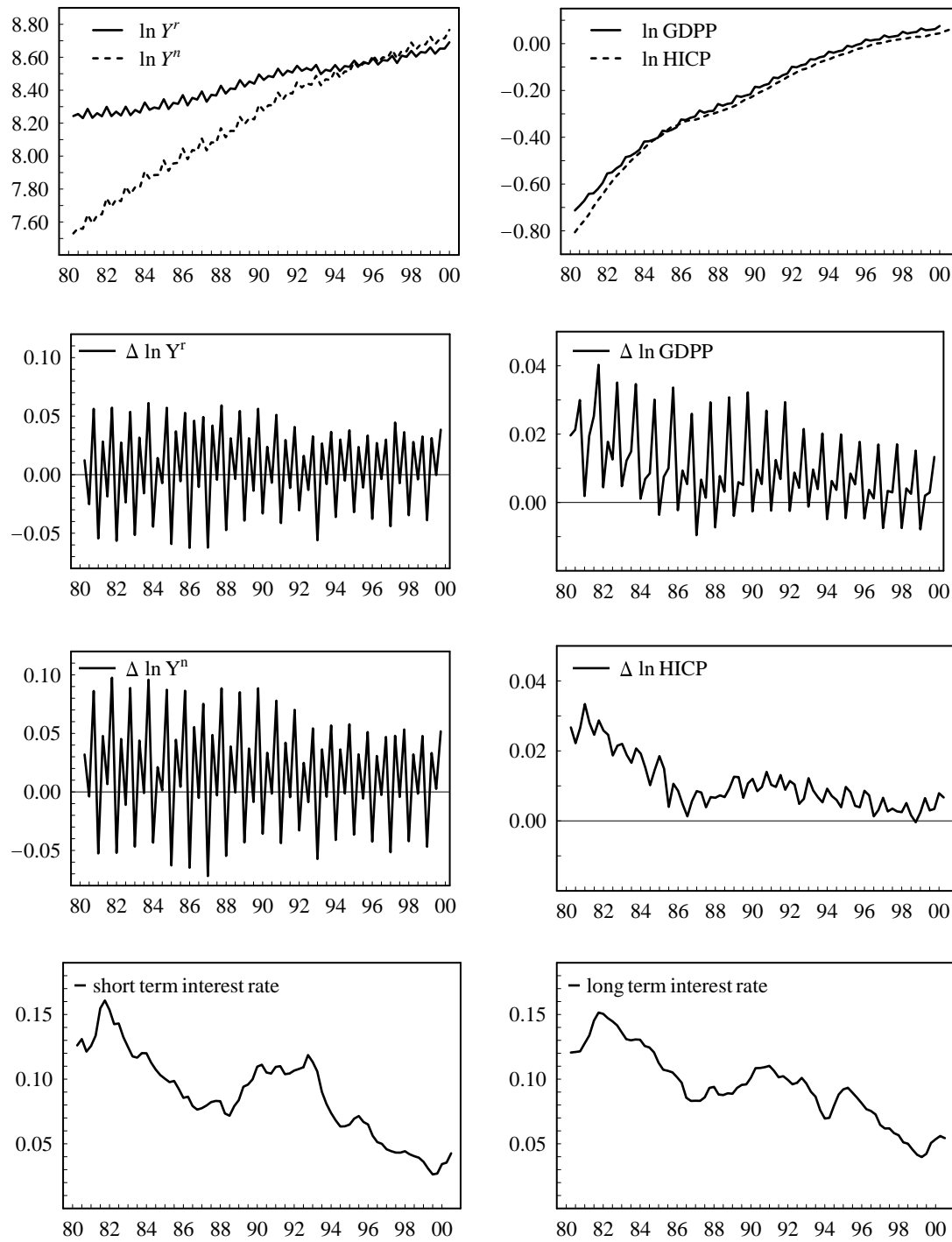


Table 7: Deterministic terms in the I(2) Model

Model	1st Step U. R.	2nd Step U. R.	Features	Critical Values
2	c		non-zero mean in both the I(0) and I(1) components, linear trend in the I(2) components	table 10 and Paruolo (1996), table 5
3.1	c	c	non-zero mean in the I(0) components, linear trend in the I(1) and I(2) components	table 11 and Paruolo (1996), table 6
3.2	c	c	non-zero mean in the I(0) components, linear trend in the I(1) components, quadratic trend in the I(2) components	table 12 and Paruolo (1996), table 13
4.1	c t	c	linear trend in the I(0), I(1) and I(2) components	table 13 and Rahbek et al. (1999), table 1

Notes: The numbering of the models corresponds to the numbering in Mosconi (1998, p. 27). U(nrestricted) and R(estricted) indicate how deterministic terms are included in the first and second step of the two-step estimator for I(2) models (c denotes a constant and t a linear trend).

Appendix B. I(2) Analysis

B.1 Implementation of the Two-Step Estimation of I(2) Models

In this section the implementation of the two-step estimation procedure for I(2) models developed by Johansen (1995b) and extended in order to include further specifications of deterministic terms by Paruolo (1996) and Rahbek et al. (1999) is summarized. For this purpose, a consistent notation covering all cases considered in the cited studies is used.

B.1.1 First Step of the Two-Step Estimation

In the first step, the adjustment parameters α and the cointegration vectors β are estimated by reduced rank regression. At this stage, the restriction on the rank of $\alpha'_\perp \Gamma \beta_\perp$ is ignored. The representation that is considered here is

$$\Delta^2 x_t = \mu_t + \Pi x_{t-1} + \Gamma \Delta x_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 x_{t-i} + u_t, \quad t = 1, \dots, T, \quad (2.1)$$

where $\Pi = \alpha \beta'$, $\mu_t = \mu_0 + \mu_1 t$ and $u_t \sim N_p(0, \Sigma_u)$. Four different specifications of the deterministic terms μ_t are taken into account here, see table 7.

The first step can be structured in the following way (an asterisk indicates that the corresponding matrices differ for different specifications of the deterministic terms):

$$Z_t = (\Delta^2 x'_{t-1}, \dots, \Delta^2 x'_{t-k+2})', \quad Z_t^* = (Z'_t, D_t^{u1'})',$$

where D_t^{u1} contains the unrestricted deterministic terms in the first step. For model 2, D_t^{u1} is empty and in the case of model 3.1, 3.2, and 4.1 an unrestricted constant is included in the first step, that is $D_t^{u1} = 1$. Further, define

$$Z_{0t} = \Delta^2 x_t, \quad Z_{1t} = \Delta x_{t-1}$$

and

$$Z_{2t} = x_{t-1}, \quad Z_{2t}^* = (Z'_{2t}, D_t^{r1'})',$$

where D_t^{r1} contains the deterministic terms that are restricted to the cointegration space in the first step, that is $D_t^{r1} = 1$ in model 2, $D_t^{r1} = t$ in model 4.1 and D_t^{r1} is empty otherwise.

2. Compute residuals:

R_{0t} : Residuals of a multivariate OLS regression of Z_{0t} on Z_t^*

R_{1t} : Residuals of a multivariate OLS regression of Z_{1t} on Z_t^*

R_{2t} : Residuals of a multivariate OLS regression of Z_{2t}^* on Z_t^*

and residual product moment matrices

$$M_{ij} = T^{-1} \sum_{t=1}^T R_{it} R_{jt}, \quad i, j = 0, 1, 2,$$

$$M_{ij.1} = M_{ij} - M_{i1} M_{11}^{-1} M_{1j}, \quad i, j = 0, 2$$

3. Solve the eigenvalue problem

$$| \lambda M_{22.1} - M_{20.1} M_{00.1}^{-1} M_{02.1} | = 0$$

for eigenvalues $1 > \hat{\lambda}_1 > \dots > \hat{\lambda}_p > 0$ and eigenvectors $\hat{V} = (\hat{v}_1, \dots, \hat{v}_p)$ such that $\hat{V}' M_{22.1} \hat{V} = I$.

4. Compute the estimators for β^* , α and Σ_u given r :

$$\hat{\beta}^* = (\hat{v}_1, \dots, \hat{v}_r), \quad \hat{\alpha} = M_{02.1} \hat{\beta}^*, \quad \hat{\Sigma}_u = M_{00.1} - \hat{\alpha} \hat{\alpha}'$$

and orthogonal complements:

$$\hat{\beta}_\perp^* = M_{22.1} (\hat{v}_{r+1}, \dots, \hat{v}_p), \quad \hat{\alpha}_\perp = M_{00.1}^{-1} M_{02.1} (\hat{v}_{r+1}, \dots, \hat{v}_p)$$

The matrix β^* is of dimension $p \times r$ for models 2, 3.1 and 3.2: $\beta^* = \beta$ and of dimensions $(p+1) \times r$ in the case of model 4.1: $\beta^* = (\beta', \beta_0)'$.

5. Compute the likelihood ratio trace statistic for hypotheses about the cointegration rank r :

$$Q_r = -T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i), \quad r = 0, \dots, p-1$$

In the second step, the matrix $\alpha'_\perp \Gamma \beta_\perp^* = \xi \eta^{*'}$ is estimated by reduced rank regression and imposing the restriction $\text{rk}(\alpha'_\perp \Gamma \beta_\perp^*) = s$.

1. Define

$$Z_t^* = (Z_t', D_t^{u2'})',$$

where D_t^{u2} contains the unrestricted deterministic terms in the second step. For model 3.1, $D_t^{u2} = 1$, otherwise D_t^{u2} is empty. Further, define

$$Z_{\alpha_\perp t} = \alpha'_\perp \Delta^2 x_t,$$

$$Z_{\beta t}^* = \beta^{*'} \Delta x_{t-1}^*,$$

where $\Delta x_{t-1}^* = \Delta x_{t-1}$ for models 2, 3.1 and 3.2 and $\Delta x_{t-1}^* = (\Delta x'_{t-1}, 1)'$ in the case of model 4.1. Define

$$Z_{\beta_\perp t} = \bar{\beta}_\perp^{*'} \Delta x_{t-1}, \quad Z_{\beta_\perp t}^* = (Z_{\beta_\perp t}', D_t^{r2'})',$$

where D_t^{r2} contains the deterministic terms that are restricted in the second step, that is $D_t^{r2} = 1$ in model 3.1 and 4.1 and D_t^{r2} is empty otherwise.

2. Compute residuals:

$R_{\alpha_\perp t}$: Residuals of a multivariate OLS regression of $Z_{\alpha_\perp t}$ on Z_t^*

$R_{\beta t}$: Residuals of a multivariate OLS regression of $Z_{\beta t}^*$ on Z_t^*

$R_{\beta_\perp t}$: Residuals of a multivariate OLS regression of $Z_{\beta_\perp t}^*$ on Z_t^*

and residual product moment matrices

$$M_{ij} = T^{-1} \sum_{t=1}^T R_{it} R_{jt}, \quad i, j = \alpha_\perp, \beta, \beta_\perp,$$

$$M_{ij.\beta} = M_{ij} - M_{i\beta} M_{\beta\beta}^{-1} M_{\beta j}, \quad i, j = \alpha_\perp, \beta_\perp$$

3. Solve the eigenvalue problem

$$| \rho M_{\beta_\perp \beta_\perp . \beta} - M_{\beta_\perp \alpha_\perp . \beta} M_{\alpha_\perp \alpha_\perp . \beta}^{-1} M_{\alpha_\perp \beta_\perp . \beta} | = 0$$

for eigenvalues $1 > \hat{\rho}_1 > \dots > \hat{\rho}_{p-r} > 0$ and eigenvectors $\widehat{W} = (\widehat{w}_1, \dots, \widehat{w}_{p-r})$ such that $\widehat{W}' M_{\beta_\perp \beta_\perp . \beta} \widehat{W} = I$.

4. Compute the estimators for η^* , ξ and $\alpha'_\perp \Sigma_u \alpha_\perp$ given s :

$$\widehat{\eta}^* = (\widehat{w}_1, \dots, \widehat{w}_s), \quad \widehat{\xi} = M_{\alpha_\perp \beta_\perp . \beta} \widehat{\eta}^*, \quad \widehat{\alpha}'_\perp \widehat{\Sigma}_u \widehat{\alpha}_\perp = M_{\alpha_\perp \alpha_\perp . \beta} - \widehat{\xi} \widehat{\xi}',$$

orthogonal complements:

$$\widehat{\eta}_\perp^* = M_{\beta_\perp \beta_\perp . \beta} (\widehat{w}_{s+1}, \dots, \widehat{w}_{p-r}),$$

$$\widehat{\xi}_\perp = M_{\alpha_\perp \alpha_\perp . \beta}^{-1} M_{\alpha_\perp \beta_\perp . \beta} (\widehat{w}_{s+1}, \dots, \widehat{w}_{p-r})$$

and additionally

$$\widehat{\beta}_1 = \overline{\widehat{\beta}}_\perp \eta, \quad \widehat{\beta}_2 = \widehat{\beta}_\perp \widehat{\eta}_\perp$$

The matrix η^* is of dimension $(p-r) \times s$ for models 2 and 3.2: $\eta^* = \eta$ and of dimensions $(p-r+1) \times s$ for models 3.1 and 4.1: $\eta^* = (\eta', \eta_0)'$.

3. Compute the likelihood ratio trace statistic for hypotheses about the cointegration index s given r :

$$Q_{r,s} = -T \sum_{s+1}^{p-r} \ln(1 - \hat{\rho}_i), \quad s = 0, \dots, p - r - 1$$

and the joint likelihood ratio trace statistic for hypotheses about the pair of cointegration indices (r, s) :

$$S_{r,s} = Q_r + Q_{r,s}$$

B.1.3 Testing Restrictions on the Cointegration Parameters

The following description of the likelihood ratio test of restrictions on the cointegration parameters follows Kongsted (1998, Chapter 3.2). The null hypothesis is that all cointegration vectors are subject to the same set of linear restrictions on the coefficients. Like in the I(1) model these restrictions can be summarized in a $p \times q$ matrix B such that $\beta = B\varphi$ and $B'_\perp \beta = 0$, where φ is of dimension $q \times r$ and q is the number of freely estimated coefficients per cointegration vector.²³ To calculate the likelihood ratio test statistic, the first step is estimated unrestrictedly as described in the previous section, and then, the first step is repeated with imposing the restriction. That is done by solving the eigenvalue problem

$$|\lambda B' M_{22.1} B - B' M_{20.1} M_{00.1}^{-1} M_{02.1} B| = 0$$

for eigenvalues $1 > \lambda_1^* > \dots > \lambda_p^* > 0$ and eigenvectors $V^* = (v_1^*, \dots, v_p^*)$. The restricted estimators are calculated as follows

$$\hat{\varphi} = (v_1^*, \dots, v_r^*), \quad \hat{\beta} = B\hat{\varphi}$$

and the likelihood ratio test statistic is

$$Q_{b1} = T \sum_{i=1}^r \ln \left(\frac{1 - \lambda_i^*}{1 - \hat{\lambda}_i} \right).$$

Q_{b1} is χ^2 -distributed with $(p - q)r$ degrees of freedom and the null hypothesis is rejected if the p -value of Q_{b1} is lower than the desired nominal significance level.

The same restrictions have also to be imposed on β_1 , the basis of the CI(2,1) relation(s). It can be shown that

$$\eta = \beta'_\perp \beta_1 = (\bar{B}\varphi_\perp, B_\perp)' \beta_1 = \begin{pmatrix} (\bar{B}\varphi_\perp)' \beta_1 \\ B'_\perp \beta_1 \end{pmatrix}$$

such that the restrictions that have to be imposed on η in the second step can be summarized by $\eta = H\kappa$ with $H' = (I_{q-r}, 0)'$, where κ contains the freely estimated coefficients and is of dimension $(q - r) \times s$. The matrix H has to be corrected if a restricted constant occurs in the second step (models 3.1 and 4.1):

$$H^* = \begin{pmatrix} H & 0 \\ 0 & 1 \end{pmatrix},$$

otherwise $H^* = H$. Now, the eigenvalue problem

$$|\rho H^{*'} M_{\beta_\perp \beta_\perp \cdot \beta} H^* - H^{*'} M_{\beta_\perp \alpha_\perp \cdot \beta} M_{\alpha_\perp \alpha_\perp \cdot \beta}^{-1} M_{\alpha_\perp \beta_\perp \cdot \beta} H^*| = 0$$

²³ The construction of the restriction matrix and the test algorithm in the I(1) model is described in Johansen (1995a, Chapter 5.3 and 7.2).

is solved for eigenvalues $1 - \rho_1^* > \dots > \rho_{p-r}^* > 0$ and eigenvectors $w = (w_1, \dots, w_{p-r})$. The likelihood ratio test statistic is

$$Q_{b2} = T \sum_{i=1}^s \ln \left(\frac{1 - \rho_i^*}{1 - \hat{\rho}_i} \right).$$

Q_{b2} is χ^2 -distributed with $(p - q)s$ degrees of freedom and the null hypothesis is rejected if the p -value of Q_{b2} is lower than the desired nominal significance level.

The restrictions on the cointegration vectors are not accepted if one of the two hypotheses in the first and second step is rejected. If the nominal significance level for both tests is $\nu/2$, respectively, the size of the joint test is between $\nu/2$ and ν .

Table 8: Critical values for the Hasza/Fuller test for double unit roots ($HF-F$)

T	(d)			(a)			(b)			(c)		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
25	2.89	3.78	6.01	5.78	7.17	10.55	9.54	11.41	15.88	17.21	20.49	28.62
50	2.82	3.60	5.52	5.47	6.61	9.22	8.75	10.17	13.43	13.16	15.13	19.43
100	2.78	3.53	5.31	5.33	6.36	8.65	8.36	9.58	12.31	11.77	13.27	16.50
250	2.76	3.49	5.20	5.25	6.23	8.36	8.13	9.25	11.70	11.07	12.40	13.25
500	2.76	3.48	5.17	5.22	6.19	8.28	8.05	9.15	11.52	10.84	12.13	14.84
∞	2.75	3.47	5.14	5.21	6.16	8.22	7.98	9.05	11.37			

Notes: Critical values for the Hasza/Fuller joint test for double unit roots described in the text. (d), (a), (b) and (c) indicate that no deterministic terms, a constant, a constant and a linear trend, or a constant, a linear trend and a quadratic trend have been added to the test regression, respectively. The critical values are taken from Hasza and Fuller (1979) for (a), (b) and (d) and from Haldrup (1994b) for (c).

Table 9: Critical values for the Sen/Dickey test for double unit roots ($SD-F$)

T	(a)			(b)			(c)		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
25	7.68	9.87	15.29	15.62	19.12	27.72	25.94	30.9	35.97
50	7.53	9.44	13.95	13.69	16.42	22.79	21.20	24.51	27.75
100	7.46	9.22	13.28	12.73	15.08	20.33	19.32	22.13	24.80
250	7.42	9.09	12.88	12.15	14.27	18.86	18.29	20.81	23.17
500							18.02	20.47	22.8
∞	7.39	9.01	12.61	11.76	13.73	17.87			

Notes: Critical values for the Sen/Dickey symmetric test for double unit roots described in the text. (a), (b) and (c) indicate that a constant, a constant and a linear trend, or a constant, a linear trend and a quadratic trend have been added to the test regression, respectively. The critical values are taken from Sen and Dickey (1987) for (a) and (b) and from Kim and Shin (1999) for (c).

Table 10: Critical values for testing hypotheses about cointegration indices (Model 2)

$p - r$	r	95% quantiles							
7	0	317.57	280.23	245.54	215.50	188.46	166.18	147.45	131.70
6	1		240.35	206.83	179.00	154.08	132.85	115.47	102.14
5	2			171.89	145.66	122.05	102.73	86.94	76.07
4	3				116.31	94.75	76.81	63.15	53.12
3	4					70.87	54.53	42.96	34.91
2	5						36.12	26.00	19.96
1	6							12.93	9.24
$p - r - s$		7	6	5	4	3	2	1	0

Notes: Critical values (95% percentiles) for testing hypotheses about the cointegration indices in the $I(2)$ model 2 (see table 7). The critical values are taken from table 5 in Paruolo (1996).

Table 11: Critical values for testing hypotheses about cointegration indices (Model 3.1)

$p - r$	r	95% quantiles							
7	0	317.57	276.86	240.72	210.71	182.30	158.63	140.14	124.24
6	1		240.35	203.12	174.83	148.54	126.69	109.21	94.15
5	2			171.89	142.57	117.63	97.97	81.93	68.52
4	3				116.31	91.41	72.99	57.95	47.21
3	4					70.87	51.35	38.82	29.68
2	5						36.12	22.60	15.41
1	6							12.93	3.84
$p - r - s$		7	6	5	4	3	2	1	0

Notes: Critical values (95% percentiles) for testing hypotheses about the cointegration indices in the I(2) model 3.1 (see table 7). The critical values are taken form table 6 in Paruolo (1996).

Table 12: Critical values for testing hypotheses about cointegration indices (Model 3.2)

$p - r$	r	95% quantiles							
7	0	307.23	269.70	234.30	205.22	178.25	155.82	136.40	124.24
6	1		231.43	197.46	169.09	144.12	123.30	105.98	94.15
5	2			163.82	139.92	113.58	94.05	78.62	68.52
4	3				108.65	87.04	69.23	55.27	47.21
3	4					63.78	47.85	36.28	29.68
2	5						30.25	19.79	15.41
1	6							5.99	3.84
$p - r - s$		7	6	5	4	3	2	1	0

Notes: Critical values (95% percentiles) for testing hypotheses about the cointegration indices in the I(2) model 3.2 (see table 7). The critical values are taken form table 13 in Paruolo (1996).

Table 13: Critical values for testing hypotheses about cointegration indices (Model 4.1)

$p - r$	r	95% quantiles							
7	0	351.57	311.22	273.99	241.23	211.55	186.08	164.64	146.77
6	1		269.24	233.77	202.76	174.94	151.27	130.93	115.35
5	2			198.22	167.91	142.15	119.83	101.47	87.15
4	3				136.98	113.04	92.24	75.30	62.76
3	4					86.66	68.23	53.19	42.66
2	5						47.60	34.36	25.43
1	6							19.87	12.49
$p - r - s$		7	6	5	4	3	2	1	0

Notes: Critical values (95% percentiles) for testing hypotheses about the cointegration indices in the I(2) model 4.1 (see table 7). The critical values are taken form table 4 in Rahbek et al. (1999).

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